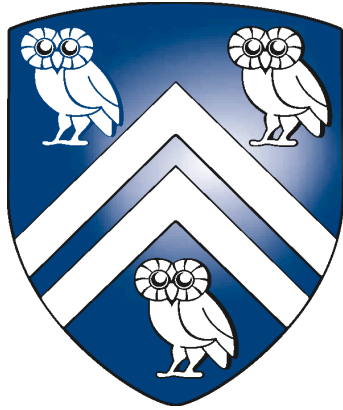




# RICE

Rice University  
MECH 590 - Aerospace Propulsion  
Design of a Hypergolic Propulsion System for a Large Lunar Lander  
Spring 2026  
Dr. Christopher Radke

Due Date: April 24, 2026

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## I. Mission and System Design

**Purpose:** It is 2100 and commercial space flight is upon us! In MECH 590, students now take a field trip to the Moon. They launch from Earth into LLO (low lunar orbit ~ 100 km above the lunar surface) via a heavy-lift launch vehicle. In this mission, we aim to send a group of four students, along with extensive lunar equipment, from LLO to the lunar surface and back using a detachable lander.

The following values were found after multiple iterations with the other sections, specifically the second and seventh sections.

**Lander Type:** Large Crew / Cargo Lander

**Payload Mass:** 20,000 kg

**Propellant:** MMH + NTO (hypergolic)

**Vacuum ISP:** 339 s (updated from a literature MMH+NTO value of 336 s [1], and then a value of 342 in section two after completing section seven)

**Propellant Mass:** ~56,000 kg

Our target orbital velocity at 100 km above the lunar surface is given by Heister Equation 2.8, where the Earth's radius is replaced by the Moon's radius:

$$v_c = \sqrt{\frac{\mu}{r_m + z}}$$

Given the following values,

$$\mu_m = 4902.8 \frac{kg^3}{s^2}, r_m = 1737 km, z = 100 km$$

Our target orbital velocity is:

$$v_c = 1633.68 \frac{m}{s}$$

To calculate  $\Delta V$ , we assume that  $2v_c$  is the base  $\Delta V$  without losses (our mission is a round trip journey), and the only loss is due to gravity, which is 20% of  $2v_c$  as a typical estimate. The upward journey, from the moon's surface to LLO, should take a larger portion of the total  $\Delta V$ , as

it is required to accelerate from rest against gravity. For the downward journey, since there is no atmosphere on the moon, any apparatuses that rely on drag are unusable, thus all maneuvers must be performed by the engine. So, a worst case scenario would be sizing the downward journey the same as the upward journey.

$$\Delta V = 2v_c + 0.2(2v_c) = 3920.84 \frac{m}{s}$$

Plugging  $\Delta V$  and ISP values into the Tsiolkovsky rocket equation and solving for MR:

$$MR = \exp\left(\frac{\Delta V}{I_{sp} \cdot g_0}\right), g_0 = 9.807 \frac{m}{s^2}$$

$$MR = 3.26$$

An inert mass of 5,000 kg is assumed (justified after calculating wet mass). Given a payload mass of 20,000 kg and inert mass assumption of 5,000 kg, we can calculate our propellant mass through the definition of MR:

$$MR = \frac{m_{payload} + m_{inert} + m_{propellant}}{m_{payload} + m_{inert}} = 3.77$$

Rearranging,

$$m_{propellant} = 56393.40 \text{ kg}$$

**Number of Tanks: 6**

The densities of MMH (fuel) and NTO (oxidizer) are:

$$\rho_{MMH} = 874 \frac{kg}{m^3}, \rho_{NTO} = 1448 \frac{kg}{m^3}$$

Using an O/F ratio by mass of 2.27 (see section two for derivation), we can calculate masses:

$$2.27m_{MMH} = m_{NTO}$$

$$m_{MMH} + m_{NTO} = m_{prop}$$

Substituting,

$$m_{MMH} = \frac{m_{prop}}{3.27}, m_{NTO} = 2.27\left(\frac{m_{prop}}{3.27}\right)$$

The volume of propellant is given by:

$$V_{prop} = \frac{m_{MMH}}{\rho_{MMH}} + \frac{m_{NTO}}{\rho_{NTO}} = 46.76 \text{ m}^3$$

Finally, a tank volume of 10 cubic meters is assumed. As a cylinder, with a radius of 1 m, the length is only around 3 m, thus it is a reasonable sized tank volume.

$$n_{tanks} = \frac{V_{prop}}{V_{tank}} = 6$$

Note: the tank volume will be less than 10 cubic meters. The assumed tank volume was used purely to determine a reasonable number of tanks.

**Wet Mass:** ~81,000 kg

Wet mass is given as the sum of inert, payload, and propellant masses.

$$m_{payload} + m_{inert} + m_{propellant} = 81393.40 \text{ kg}$$

The inert mass assumption of 5,000 kg is approximately 6% of the total wet mass. This value is much lower than the Apollo Lunar Module during its descent over a span of missions [2], which was around 25%. Refer to Table 1, which was pulled from the hyperlinked source, for specific values. However, this mission's 6% assumption can be validated by the large payload requirement, which drives up propellant mass. Also, this mission uses a large cargo lander, whereas the Apollo Lunar Module was small and designed for only two people.

| Mission:                    | 11     | 12     | 13     | 14     | 15     | 16     | 17     |
|-----------------------------|--------|--------|--------|--------|--------|--------|--------|
| <b>Descent Stage</b>        |        |        |        |        |        |        |        |
| <i>Inert Mass (kg)</i>      | 4,483  | 4,875  | 4,650  | 4,712  | 6,179  | 6,083  | 6,155  |
| <i>Propellant Mass (kg)</i> | 18,184 | 18,429 | 18,435 | 18,417 | 19,562 | 19,559 | 19,564 |
| <i>PMF</i>                  | 0.8022 | 0.7908 | 0.7986 | 0.7963 | 0.7600 | 0.7628 | 0.7607 |
| <b>Ascent Stage</b>         |        |        |        |        |        |        |        |
| <i>Inert Mass (kg)</i>      | 4,804  | 4,760  | --     | 4,691  | 4,690  | 4,704  | 4,729  |
| <i>Propellant Mass (kg)</i> | 5,238  | 5,236  | --     | 5,225  | 5,237  | 5,243  | 5,262  |
| <i>PMF</i>                  | 0.5216 | 0.5238 | --     | 0.5269 | 0.5276 | 0.5271 | 0.5267 |

Table 1. Apollo Lunar Module Mass Breakdown

## II. Liquid Engine Design Overview

**Overview:** The engine to be used in this mission will be helium gas pressure fed. This decision was made due to the requirements of the mission. Pressure fed engines are simpler, and given a low chamber pressure (listed below), doesn't require tank walls to be unreasonably thick; since our tank pressure must be higher than our chamber pressure for propellant flow, the tank pressure will also be reasonably low. Also, pressured fed systems are much more reliable for multiple engine restarts, as a pump fed system would require turbomachinery spin-ups, with many more spots for potential misfires. The engine will feature a de Laval nozzle, a crucial aspect of the engine which enables supersonic propellant flow. The engine is also to be throttled during landing.

**Chamber Pressure:** 100 psia (689.476 kPa)

This value was selected after extensive combustion analysis under equilibrium assumption, utilizing NASA's CEA MATLAB wrapper. The propellant is to be stored at room temperature: 298.15 K (section three discusses this further). Initially, simulations were run for a range of chamber pressure values spanning from 100 to 200 psia, which are reasonable chamber pressures (see section nine's benchmark data). The following properties were stored and plotted against chamber pressure:

- (1)  $C^*$
- (2) vacuum ISP
- (3) mass flow per engine
- (4) throat area
- (5)  $C_f$
- (6) burn time

See Figure 1 below:

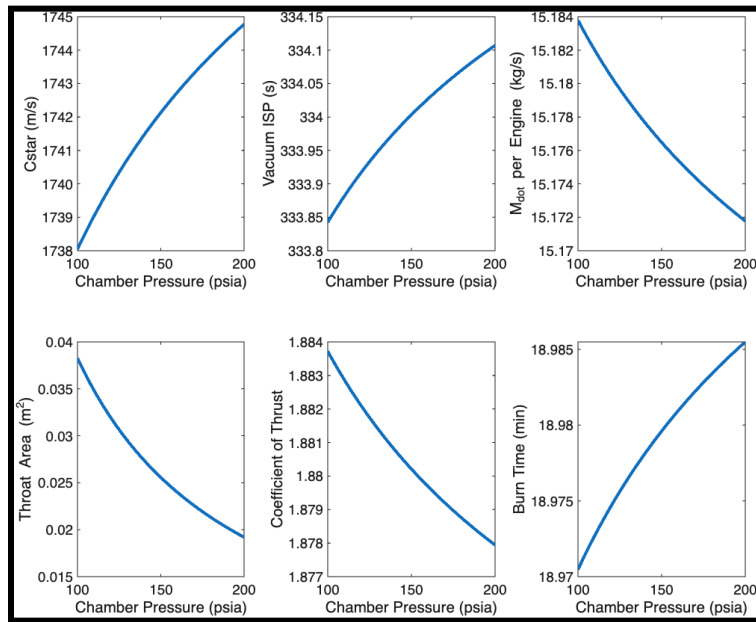


Figure 1. Various Properties vs. Chamber Pressure

These properties didn't appear to vary much with chamber pressure. To confirm this observation, a contour map showing how vacuum ISP varied with O/F ratio and chamber pressure was created. See Figure 2.

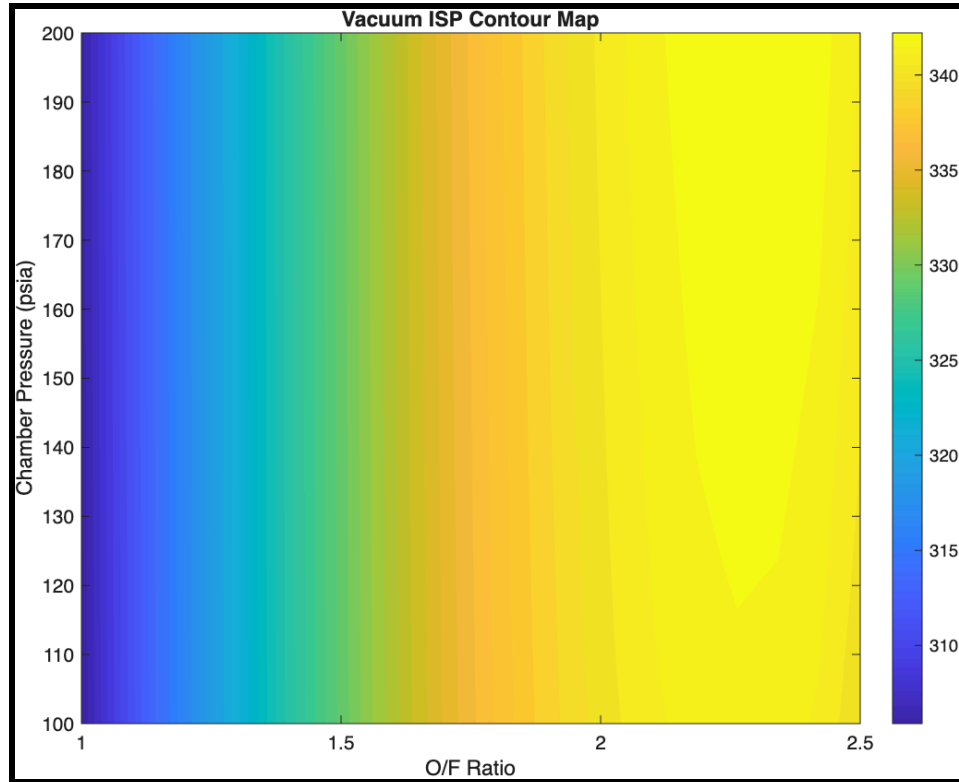


Figure 2. Contour Plot of Chamber Pressure and O/F Ratio Effects on Vacuum ISP

Figure 2 emphasizes the importance of O/F ratio over chamber pressure, as the color gradient seems to have almost no change going up in chamber pressure value. So, in order to minimize weight, the chamber pressure was minimized at 100 psia.

**O/F Ratio: 2.27**

This value was selected, like chamber pressure, from equilibrium combustion analysis. CEA was used to create another set of plots of the same properties, but instead swept over O/F ratios from 1 to 2.5. See Figure 3.

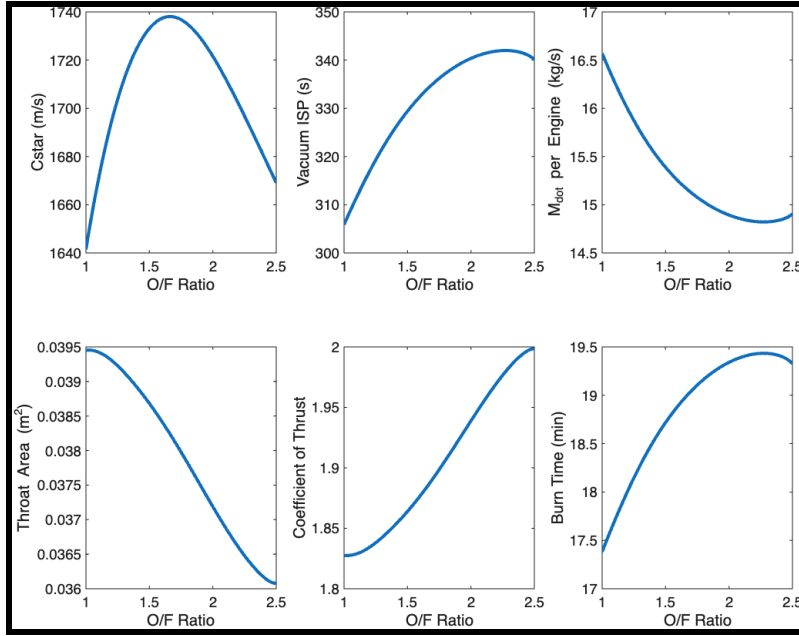


Figure 3. Various Properties vs. O/F Ratio

Figure 3 shows a clear optimum point for vacuum ISP, burn time, and C\*. We will take the maximizing O/F ratio for ISP (hence, also burn time) instead of C\* because ISP is a metric evaluating the total performance of the engine, whereas C\* measures specifically the combustion performance. The O/F ratio that yielded the max ISP value was 2.27, and the max ISP was 342 s (this ISP value was updated to 339 s after completing heat transfer analysis in section seven).

**Maximum Thrust:** ~43 kN per engine

Heister Equation 3.16, with acceleration on Earth's surface replaced with that on the Moon's surface, states that the thrust-to-weight ratio must be greater than one:

$$\eta_0 = \frac{F}{m_o g_m} > 1$$

A ratio of 1.2 is chosen from a typical range of 1.1 to 1.5. Solving for total thrust:

$$F = \eta_0 * m_o g_m > 1, m_o = m_{wet}, g_m = 1000 * \frac{\mu_m}{r_m^2}$$

$$F = F_{total} = 171940.07 N$$

The lander will utilize four engines for efficient and effective throttling, so maximum thrust per engine is:

$$F_{engine} = \frac{F_{total}}{n_{engines}} = 42985.02 \text{ N}$$

**Minimum Thrust:** ~18 kN per engine

Now, the thrust-to-weight ratio is set to one in Heister Equation 3.16, in order to calculate minimum thrust. This value is used during landing in order to perform a safe, hover maneuver.

$$F_{total, min} = \eta_0 * m_o g_m = 73300.87 \text{ N}$$

$$F_{engine, min} = \frac{F_{total}}{n_{engines}} = 18325.22 \text{ N}$$

**Throttling:** 43% of max thrust

In order to figure out throttling for this engine, the ratio of minimum to maximum thrust was calculated, and the value was 0.43. The four engine configuration allows for minimum throttling in each engine: two engines are shut off to reach 50% of max thrust, then the two remaining engines are throttled to 86%, effectively reaching a total of 43% of max thrust. We want to minimize throttling because it could affect the film cooling (see section seven) capabilities of the engine. By decreasing the mass flow rate for throttling, it could cause the film to detach from the engine wall, causing overheating and possible thermal failure.

**Vacuum ISP:** 339 s

This ISP value was found in section seven.

**C\*:** 1694 m/s

Characteristic velocity was extracted from its respective plot in Figure 3 at the ISP-maximizing O/F ratio.

**Mass Flow Rate per Engine:** 12.94 kg/s

The total mass flow rate is given by

$$m_{dot} = \frac{F_{total}}{ISP * g_0} = 51.76 \text{ kg/s}$$

To find the mass flow rate per engine, simply divide the total mass by the number of engines:

$$m_{dot, engine} = \frac{m_{dot}}{n_{engines}} = 12.94 \text{ kg/s}$$

**Throat Area:** 0.0318 m<sup>2</sup>

First, convert chamber pressure from psia to Pa by multiplying the value in psia by 6894.76.

Then, the throat area is given by:

$$A_t = \frac{m_{\dot{engine}} C^*}{P_{c, Pa}}$$

**Coefficient of Thrust:** 1.96 m/s

The equation for the coefficient of thrust is

$$C_f = \frac{F_{engine}}{P_{c, Pa} A_t} = 1.96$$

**Nozzle Expansion Ratio:** 50:1

This expansion ratio is justified in section nine's benchmarking data, where similar expansion ratios are utilized for engines firing into vacuum. Since this mission takes place purely on the Moon, the firing conditions will purely be in a vacuum, and our nozzle will always be under-expanded. In order to get as close to optimum expansion as possible, we must maximize the expansion ratio; that is, without creating unreasonable mass effects. Use Figure 2, which is derived from homework 2 problem 2 for a specific nozzle configuration, as reference for expansion ratio effects. The numeric values are irrelevant; rather, focus on the trends of each plot.

**Total Burn Time:** ~18 min

For this first order calculation, we assume that all propellant is utilized. First, we calculate the mass flow rate, then divide propellant mass by the total mass flow rate to get the total burn time:

$$t_{burn} = \frac{m_{prop}}{m_{\dot{engine}}} = 18.16 \text{ min}$$

### III. Pressurization / Propellant

#### **Engine Architecture:** Pressure fed

As previously stated, the engine is helium gas pressure fed. The low chamber pressure of 100 psia doesn't require unreasonably thick tank walls. Since propellant tank pressure must exceed chamber pressure to drive flow, the tank pressure will also remain low.

#### **Pressurant:** Helium

Helium is the industry standard for pressure fed rocket systems. It is chemically inert with both MMH and NTO, thus it doesn't affect engine performance.

#### **Pressure Budget:** 30% of chamber pressure

The pressure budget is established working backwards from the chamber. The propellant tank pressure must exceed chamber pressure to overcome all losses between the tank and the combustion chamber. These losses come from three sources: feed lines, valves and fittings, and the intentional pressure drop across the injector orifices. The dominant loss term is the injector pressure drop, which is intentionally sized at 20% of chamber pressure. From Huzel Chapter IV: "The rule-of-thumb design value for injector pressure drop varies from 15 to 20 percent of the chamber-nozzle stagnation pressure." The upper-end value of 20% was chosen to ensure sufficient velocity for propellant atomization and mitigate potential flow reversal. An additional 10% of chamber pressure is allocated to losses due to the feed lines, valves, and fittings. This value is treated as a design margin, since detailed hydraulic analysis of the feed system is beyond the scope of this work. So, using the total pressure budget of 30%, the propellant can be pressurized accordingly.

$$P_{tank, prop} = P_c + \Delta P_{injector} + \Delta P_{lines} + \Delta P_{valves, fittings} = 1.3 * P_c = 130 \text{ psia}$$

#### **Required Helium Mass:** 67.68 kg

Helium is stored at 4,500 psia. It is similar to the X-34 Main Propulsion System, which housed helium at 5000 psia [3]. Assuming that helium behaves as an ideal gas — in fact, it is a real gas that exhibits the most similar behavior to an ideal gas, as it is a monatomic noble gas [4]— the ideal gas law can be used to find the required helium mass for this mission. Once all propellant has been expelled, helium must occupy the total propellant volume at feed pressure.

$$m_{He} = P_{tank, prop} * V_{prop} \div (R_{He} * T_{Prop})$$

The R constant for Helium is found from

$$R_{He} = \frac{R}{MW_{He}}, R = 8.314 \text{ J/mol K}, MW_{He} = 0.0040026 \text{ kg/mol}$$

$$R_{He} = 2077.1 \text{ J/kgK}$$

Plugging back into the equation to find helium mass,

$$m_{He} = 67.68 \text{ kg}$$

**Helium Tank:** Spherical, Ti-6Al-4V

|                  |                   |
|------------------|-------------------|
| <b>Pressure</b>  | 4500 psia         |
| <b>Volume</b>    | 1.35 cubic meters |
| <b>Thickness</b> | 17.71 mm          |
| <b>Diameter</b>  | 2.01 m            |

Table 2. Helium Tank Specifications

A Ti-6Al-4V spherical helium tank is standard across many pressure-fed missions [3]. The volume was calculated using the ideal gas law:

$$V_{tank, He} = m_{He} * R_{He} * T_{Prop} / P_{He, Pa} = 1.35 \text{ m}^3$$

The radius was calculated from using the spherical volume formula, and using the hoop stress formula for a sphere, the thickness was calculated:

$$t_{tank, He} = P_{He, Pa} * r_{He, tank} / (2 * \sigma_y), \sigma_y = 880 \text{ MPa} = 880 * 10^6 \text{ Pa}$$

$$t_{tank, He} = 17.71 \text{ mm}$$

### **Propellant Considerations:** MMH and NTO

As previously mentioned, the propellants are stored at 298.15 K, as they remain liquid at ambient conditions. However, these propellants are highly toxic and corrosive, requiring numerous safety precautions. Also, temperatures on the moon can drop to absolute zero [5]. Given the propellants have melting points much higher than 0 K, they must be thermally managed throughout the mission to ensure they stay liquid.

#### IV. Injector Design

|                                  | <b>Main</b>   | <b>Film Cooling</b> |
|----------------------------------|---------------|---------------------|
| <b>MMH Orifice Diameter</b>      | 2.97 mm       | 2.25 mm             |
| <b>NTO Orifice Diameter</b>      | 3.94 mm       | 1.40 mm             |
| <b>MMH Orifice Angle</b>         | 29.04 degrees | 12.16 degrees       |
| <b>NTO Orifice Angle</b>         | 15.96 degrees | 32.84 degrees       |
| <b>Distance Between Orifices</b> | 9 mm          | 9 mm                |
| <b>Impingement Distance</b>      | 10.71 mm      | 10.46 mm            |
| <b>Number of Doublets</b>        | 50            | 10                  |

Table 3. Injector Specifications

The design is an unlike doublet impinging element injector. According to Heister 8.6.2, since hypergolics ignite spontaneously on contact, this type of design is optimal due to large interfacial contact. In addition, since MMH and NTO have similar volume flows, doublets, as opposed to triplets or other groupings, are ideal. The injector contains orifices for both main flows and film cooling flows. The material to be used is Inconel 625, an alloy with superior performance at high temperature and pressure. It resists oxidation and fracture, which are two critical factors to look at when designing an injector that contains oxidizer flows through sharp holes, where stress concentrations can build up [6]

To begin the design process, mass flow per engine was broken up into fuel and oxidizer flows:

$$m_{\dot{dot}, engine, MMH} = m_{\dot{dot}, engine} / (1 + O/F) = 3.95 \text{ kg/s}$$

$$m_{\dot{dot}, engine, NTO} = (m_{\dot{dot}, engine} * O/F) / (1 + O/F) = 8.99 \text{ kg/s}$$

The mass flows were further divided into film and main flows, based on a fuel mass percentage assumption of 5% going to film cooling (justified in section seven). The generic equations used were:

$$m_{\dot{dot}, film} = 0.05 * m_{\dot{dot}}$$

$$m_{\dot{dot}, main} = 0.95 * m_{\dot{dot}}$$

The mass flows were once more divided into specific orifice flows, for both main and film cooling orifices. The generic equation used was:

$$m_{dot, orifice} = m_{dot} / n_{orifices}$$

Orifice areas were calculated using Heister Equation 8.25:

$$m_{dot} = C_D A \sqrt{2\rho g \Delta p} \rightarrow A = m_{dot} / (C_D \sqrt{2\rho g \Delta p})$$

$$\Delta p = \Delta P_{injector}, \rho = \rho_{MMH \text{ or } NTO}, C_D = 0.7$$

The discharge coefficient,  $C_D$ , is set to 0.7 as an assumption made based on Heister's range of 0.6 to 0.9. A value of 0.7 indicates that the orifices are fairly sharp, chosen for manufacturing simplicity. Diameters were calculated from a circular orifice area as specified in Table 2. To calculate orifice angles, Heister Equation 8.29 yields:

$$m_{dot, f} v_f \sin \alpha_f = m_{dot, ox} v_{ox} \sin \alpha_{ox}$$

Refer to Heister Figure 8.21 (renamed Figure 4 for naming convention continuity):

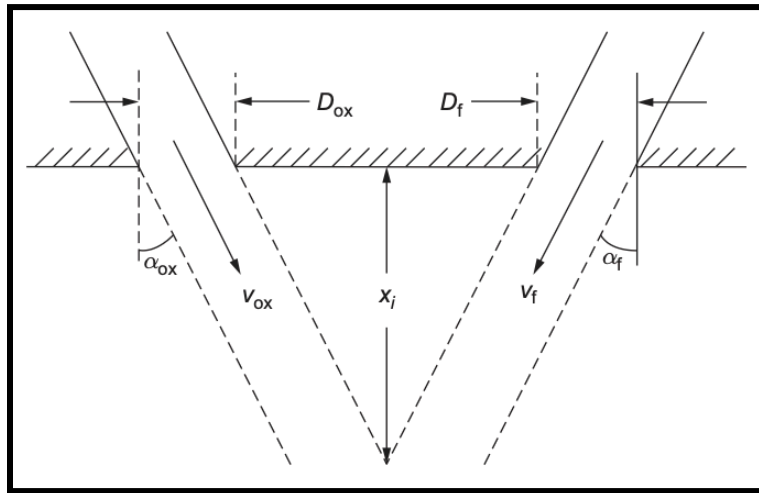


Figure 4. Geometry for an Unlike Doublet Impinging Element Design

. Heister recommends that the sum of the angles lie between 45 and 60 degrees. A value of 45 degrees was selected to maximize the impinging distance for a given distance between the two orifices in the doublet. To solve for specific angles, first, a sine ratio was calculated using a rearrangement of Heister Equation 8.29:

$$\frac{\sin \alpha_f}{\sin \alpha_{ox}} = (m_{dot, ox} v_{ox}) / (m_{dot, f} v_f) = \sin_{ratio}$$

With the sine ratio defined as a numeric value, two equations were used:

$$\Sigma\alpha = \alpha_f + \alpha_{ox}$$
$$\sin_{ratio} = \frac{\sin\alpha_f}{\sin\alpha_{ox}}$$

Combining these equations, the two angles can be solved for. Trigonometric identities were used to yield:

$$\alpha_{ox} = \tan^{-1} [ \sin(\Sigma\alpha) / (\sin_{ratio} + \cos(\Sigma\alpha)) ]$$
$$\alpha_f = \Sigma\alpha - \alpha_{ox}$$

This set of equations was used to calculate main and film cooling angles for NTO and MMH as shown in Table 2. The distance between orifices in a single doublet, measured from center to center, for both main and film cooling doublets, were set to 9 mm, allowing impingement distances to be at least 10 mm, minimizing the chances of potential splashback.

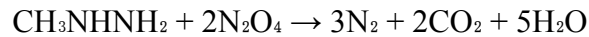
## V. Combustion Modeling

### **Combustion Modeler:** Chemical Equilibrium with Applications (CEA)

As seen in section two and section seven, NASA's CEA software was used to run several simulations, calculating local properties and serving as a reference for engine performance with varying parameters. For code integration, the CEA MATLAB wrapper was used, enabling fast sweeps of parameters, with outputs saved locally as variables.

CEA solves the chemical equilibrium problem by minimizing Gibbs free energy of the product mixture at specified chamber conditions. For this analysis, an equilibrium assumption was applied throughout the nozzle, meaning combustion products are assumed to continuously re-equilibrate as they expand through the nozzle. This is a standard assumption for preliminary design, as it tends to slightly overpredict ISP compared to frozen flow, where composition is fixed at the throat. This allows for conservative engine sizing.

### **Stoichiometric Combustion Reaction:**



### **Combustion Properties:**

|                                                 |               |
|-------------------------------------------------|---------------|
| <b>Specific Heat Ratio, <math>\gamma</math></b> | 1.13          |
| <b>Molecular Weight</b>                         | 22.68 kg/kmol |
| <b>Adiabatic Flame Temp</b>                     | 3144.97 K     |

Table 4. Combustion Properties

The properties in Table 3 were calculated using CEA at the design point of  $\text{O/F} = 2.27$  and  $P_c = 100$  psia. The adiabatic flame temperature is taken as the chamber temperature under the assumption that there is no heat loss to surroundings, hence adiabatic. Also, complete chemical equilibrium is assumed in the chamber before expansion, consistent with the equilibrium flow assumption applied throughout the CEA analysis. Under these assumptions, all chemical energy of the MMH/NTO reaction is converted to thermal energy of the product gases, and the resulting adiabatic flame temperature directly equals the chamber temperature. Molecular weight and specific heat ratio values were calculated at the chamber, where combustion products have reached full chemical equilibrium.

As stated previously, the vacuum ISP calculated using this configuration was 342 s. The actual vacuum ISP of 339 s reflects real losses from film cooling. Specifically, diverting 5% of MMH/NTO mass flow to the chamber walls reduces the effective O/F ratio in the core stream and removes energy that would otherwise contribute to thrust.

## VI. Chamber + Nozzle Design

### Chamber Design:

Initially, the characteristic length,  $L^*$ , of the chamber was set to 35 inches, selected from the range recommended for NTO/hydrazine-base fuel, per Huzel Table 4-1 (renamed Table 4). The upper bound value was selected for worst case scenario weight consequences.

| Propellant combination                                               | Combustion chamber characteristic length ( $L^*$ ), in. |
|----------------------------------------------------------------------|---------------------------------------------------------|
| Chlorine trifluoride/hydrazine-base fuel . .                         | 30-35                                                   |
| Liquid fluorine/hydrazine . . . . .                                  | 24-28                                                   |
| Liquid fluorine/liquid hydrogen ( $\text{GH}_2$ injection) . . . . . | 22-26                                                   |
| Liquid fluorine/liquid hydrogen ( $\text{LH}_2$ injection) . . . . . | 25-30                                                   |
| Hydrogen peroxide/RP-1 (including catalyst bed) . . . . .            | 60-70                                                   |
| Nitric acid/hydrazine-base fuel . . . . .                            | 30-35                                                   |
| Nitrogen tetroxide/hydrazine-base fuel . .                           | 30-35                                                   |
| Liquid oxygen/ammonia . . . . .                                      | 30-40                                                   |
| Liquid oxygen/liquid hydrogen ( $\text{GH}_2$ injection) . . . . .   | 22-28                                                   |
| Liquid oxygen/liquid hydrogen ( $\text{LH}_2$ injection) . . . . .   | 30-40                                                   |
| Liquid oxygen/RP-1 . . . . .                                         | 40-50                                                   |

Table 5. Recommended Combustion Chamber Characteristic Length for Various Propellant Combinations

A 2.5 contraction ratio, or CR, was used, based on multiple examples from Huzel (see Chapter III) and resizing for this specific design. From  $L^*$  and CR, the combustion chamber can be fully defined.

|                              |                 |
|------------------------------|-----------------|
| Characteristic Length, $L^*$ | 35 in = 88.9 cm |
| Contraction Ratio            | 2.5             |
| Chamber Length               | 35.56 cm        |
| Chamber Diameter             | 31.82 cm        |

Table 6. Combustion Chamber Specifications

The area of the chamber is the product of CR and throat area. From there, the chamber diameter was calculated assuming a perfect cylindrical chamber. The chamber length was found by dividing the chamber volume, which is the product of  $L^*$  and throat area, by the chamber area.

### Nozzle Design: 80% Bell Nozzle Design

The 80% bell nozzle achieves equivalent thrust as a simple 15 degree conical nozzle but in only 80% of the length. The nozzle was calculated and designed using a Rao parabolic approximation via complex parametrization. A specific outline [7] was followed, starting from the contraction of the chamber, continuing through the throat and length of the nozzle.

For the entrant section:

$$\begin{aligned} x &= 1.5 R_t \cos \theta \\ y &= 1.5 R_t \sin \theta + 1.5 R_t + R_t \end{aligned} \quad \text{equ.s 4}$$

where:  $-135 \leq \theta \leq -90$

Figure 5. Initial contraction parametric equations

For the exit section:

$$\begin{aligned} x &= 0.382 R_t \cos \theta \\ y &= 0.382 R_t \sin \theta + 0.382 R_t + R_t \end{aligned} \quad \text{equ.s 5}$$

where:  $-90 \leq \theta \leq (\theta_n - 90)$

Figure 6. Final contraction parametric equations

$$\begin{aligned} x(t) &= (1-t)^2 N_x + 2(1-t)t Q_x + t^2 E_x & 0 \leq t \leq 1 \\ y(t) &= (1-t)^2 N_y + 2(1-t)t Q_y + t^2 E_y & 0 \leq t \leq 1 \end{aligned} \quad \text{equ.s 6}$$

Figure 7. Bezier Curve Parametric equations for the Bell Nozzle

Equations 6 are defined by points N, Q, and E (see the graphical method earlier for the locations of these points).

Point N is defined by equations 5 setting the angle to  $(\theta_n - 90)$ .

Coordinate  $E_x$  is defined by equation 3, and coordinate  $E_y$  is defined by equation 2.

Point Q is the intersection of the lines:  $\overline{NQ} = m_1x + C_1$  and:  $\overline{QE} = m_2x + C_2$  **equ.s 7**

where: gradient  $m_1 = \tan(\theta_n)$  , gradient  $m_2 = \tan(\theta_e)$  **equ.s 8**

and: intercept  $C_1 = N_y - m_1N_x$  , intercept  $C_2 = E_y - m_2E_x$  **equ.s 9**

The intersection of these two lines (at point Q) is given by:

$$Q_x = \frac{(C_2 - C_1)}{(m_1 - m_2)}, Q_y = \frac{(m_1 C_2 - m_2 C_1)}{(m_1 - m_2)} \quad \text{equ.s 10}$$

Figure 8. Required Points for the Bezier Curve Parametric Equations

Following the procedure highlighted in Figures 5-8 will yield an 80% bell nozzle. However, two values must be defined prior to finding Bezier curve coordinates:  $\theta_n$  and  $\theta_e$  . Using Huzel Figure 4-14 (renamed Figure 9 below), the values can be found by looking at the line that corresponds with an 80%  $L_f$  value and an  $\epsilon$  value of 50.

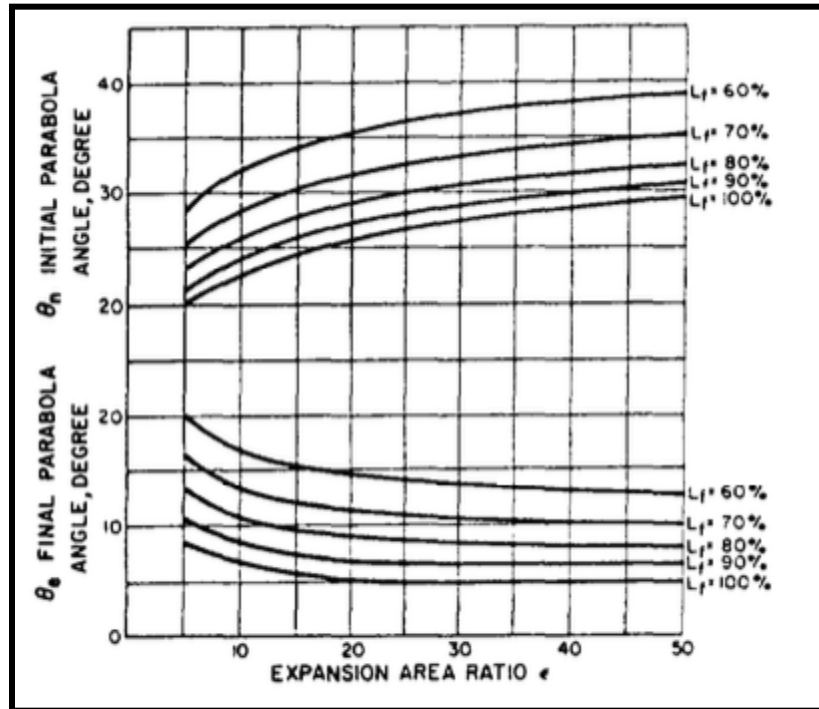


Figure 9.  $\theta_n$  and  $\theta_e$  as a Function of Expansion Area Ratio  $\epsilon$

$$\theta_n = 32.5 \text{ degrees}$$

$$\theta_e = 7.5 \text{ degrees}$$

The chamber coordinates were found by setting the endpoint to the first value of the initial contraction series, followed by a constant chamber radius value.

### Final Design:

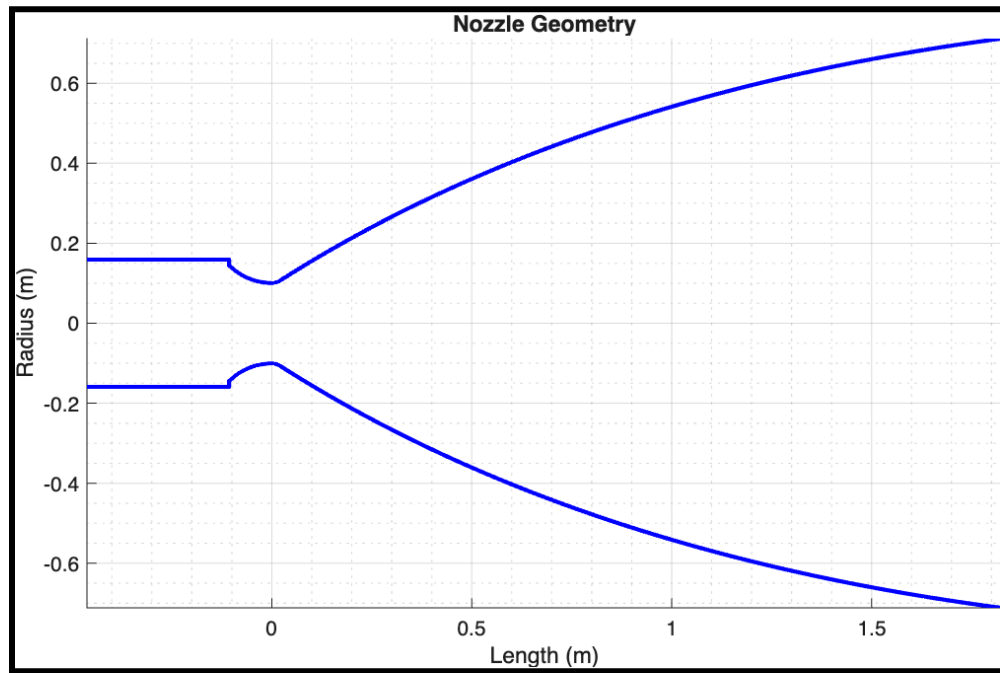


Figure 10. Chamber + Nozzle Design

Figure 10 shows all of the coordinates plotted. The bottom half of the design was plotted by setting all y-coordinates to a negative value. CEA simulations were run for the length of the chamber and nozzle, swept along local area ratios. Plots for local mach, pressure, temperature, and velocity values (found using the product of mach and sonic velocity) were created, shown in Figure 11.

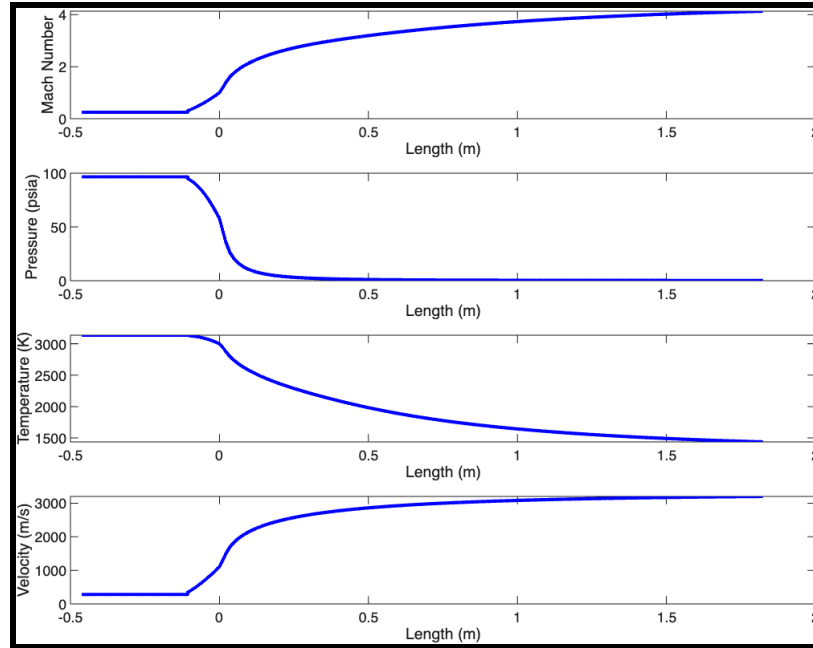


Figure 11. Local Properties vs. Area Ratios Along the Length of the Chamber and Nozzle

Finally, hoop stress calculations were performed along the length of the chamber and nozzle.

Using a thin wall assumption:

$$\sigma_{\theta} = \frac{Pr}{t}$$

Local pressure and radius values are already defined. The thickness of the chamber and nozzle,  $t$ , is set at 1 mm after multiple iterations of stress calculations. After completing the heat transfer analysis, the materials chosen for the chamber and nozzle are to be carbon phenolic from the chamber up to a nozzle expansion ratio of 20, and a nozzle skirt to be made of R512E slurry silicide coated C-103 columbium from the expansion ratio of 20 to 50. Further justifications are provided in section seven.

Hoop stress was calculated and plotted against the length of the chamber and nozzle.

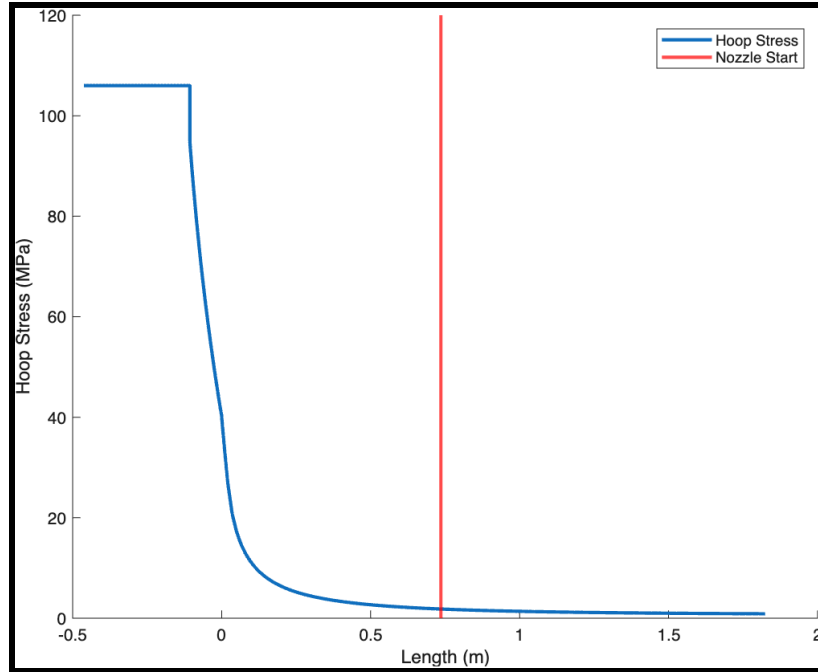


Figure 11. Hoop Stress vs. Length

The tensile strengths of carbon fiber and C-103, which are 3500 [8] and 65.5 [9] MPa (at 1480°C, a high temperature assumption and the estimated failure temperature of the skirt, explained in section seven), respectively, are much higher than the local hoop stresses, thus the design is validated. The thickness of 1 mm minimizes weight, keeps hoop stresses under tensile strengths, and remains manufacturable. However, this isn't taking into account thermal failure, which is a much more critical metric that will be investigated in the next section.

## VII. Heat Transfer Analysis

### Overview:

The heat transfer analysis section primarily determines the required cooling system for the chamber and nozzle walls. In addition, it establishes the true delivered vacuum Isp of 339 s, which accounts for the performance penalty of film cooling, which was fed back into sections one and two to update the design.

### Heat Transfer Method: Bartz Correlation

To find the wall temperatures, two functions (adapted from homework four) are used — one for the chamber and one for the nozzle. In both functions, convective heat transfer from the hot combustion gases to the chamber and nozzle walls is computed using the Bartz equation, which gives the local heat transfer coefficient as a function of local flow conditions:

$$h_g = \frac{0.026}{D^{0.2}} \cdot \frac{C_p \mu^{0.2}}{Pr^{0.6}} \cdot (\rho v)^{0.8} \cdot \frac{\rho_{am}}{\rho} \cdot \left(\frac{\mu_{am}}{\mu_0}\right)^{0.2}$$

Where D is local diameter,  $\mu$  is local viscosity,  $\mu_0$  is chamber viscosity, Pr is Prandtl number,  $\rho$  is local density, v is local velocity,  $\rho_{am}$  is arithmetic mean density, and  $\mu_{am}$  is arithmetic mean viscosity. The last two variables are calculated within a for loop containing the iterative solver, after an initial wall temperature guess of local temperature minus 10 K. They are given by:

$$\rho_{am} = 0.5\rho\left(1 + \frac{T}{T_w}\right)$$
$$\mu_{am} = (\mu + \mu_w) / 2, \mu_w = \mu \cdot \left(\frac{T_w}{T}\right)^{0.68125}$$

The recovery temperature, or the effective temperature the wall sees from the high-velocity gas, and recovery constant are computed as:

$$T_r = T \cdot \left(1 + \frac{\gamma-1}{2} r M^2\right), r = Pr^{1/3}$$

The wall temperature is finally calculated using a Newton-Raphson iteration method by iterating on the heat flux balance between convective heating from the gas and heat removal through the wall material:

$$h_g(T_r - T_w) = q_{removal}$$

This balance is set to a function  $f$ , and the Newton-Raphson method is applied to converge on a wall temperature value once the difference between the new wall temperature and previous wall temperature is less than 0.1 K:

$$f(T_w) = h_g(T_r - T_w) - q_{removal}$$

$$T_{w,new} = T_w - 0.5 \cdot f(T_w) / f'(T_w)$$

The damping factor of 0.5 was applied to prevent overshoot during iteration. The wall temperature is bounded below by 300 K and above by the local recovery temperature, since physically, the wall cannot be hotter than the adiabatic wall temperature.

In the function for the nozzle, or more specifically, the nozzle skirt, heat is removed by radiation to the surroundings at 0 K from the outer wall surface:

$$q_{removal} = \varepsilon \sigma T_w^4$$

Where  $\varepsilon = 0.7$ , the lower bound emissivity of the C-103 columbium alloy [10].

Also incorporated into the nozzle function is an actual ISP solver, which calculates the mass fractions of mass flow in the core and film to the total mass flow in the engine. The true vacuum ISP is given by:

$$ISP_{true} = m_{frac,core} * ISP_{core} + m_{frac,film} * ISP_{film}$$

Where core ISP is at the design O/F ratio.

In the chamber, heat is removed by conduction through the two-layer ablative wall — phenolic resin charring layer over carbon fiber substrate. The assumption is made that the ablative phenolic resin layer's deterioration has no effect on engine performance. The equation is derived from a conduction thermal resistor network:

$$q_{removal} = (T_w - T_{ext}) / (R_{phenolic} + R_{carbon}) = (T_w - T_{ext}) / (\frac{t_{phenolic}}{k_{phenolic}} + \frac{t_{carbon}}{k_{carbon}})$$

Where  $T_{ext} = 400$  K, the outer wall temperature. The thermal conductivity,  $k$ , values for carbon fiber and phenolic resin are 9 [11] and 0.2 [12] W/mK, respectively. The thickness of each, respectively, are 3.3 and 1 mm. The 3.3 mm is the thickness of a ten ply phenolic laminate [13].

The wall temperature calculated is the theoretical surface temperature of the phenolic resin layer. However, the true temperature of the wall, which is at the carbon fiber substrate surface, is given by:

$$T_{w, actual} = T_w - q_{cond} \cdot \frac{t_{phenolic}}{k_{phenolic}}, \quad q_{cond} = q_{removal}$$

The equation is applied after the solver converges on a value.

### Results:

The functions were run for various film cooling O/F ratios, swept over various area ratios. For the chamber, the ratios spanned from 2.5, which is the contraction ratio, to 1. For the nozzle, the ratios spanned from 1 to 50.

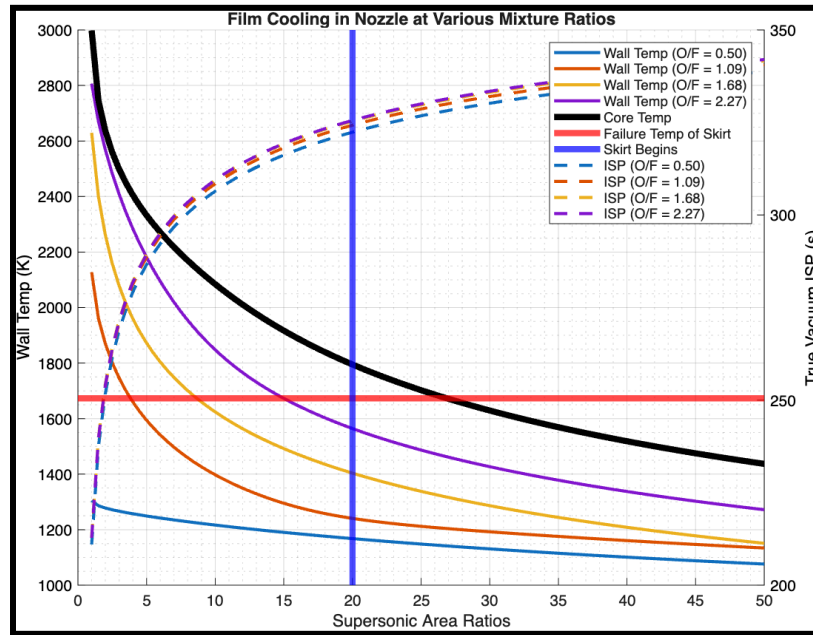


Figure 12. Film Cooling in Nozzle at Various Mixture Ratios

Figure 12 was crucial in determining the materials and specifications of the engine. The assumption is made that film cooling loses effectiveness by the time the nozzle skirt is attached, modeled by the following equation [14]:

$$\eta = \frac{T_w - T_c}{T_2 - T_c}$$

Where  $T_2$  is the injector temperature,  $T_c$  is the local gas temperature of the film cooling stream, and  $T_w$  is the calculated adiabatic wall temperature. At an area ratio of 20, the efficiency is

1.68%, so we can safely assume that there is no more film cooling and the cooling in the skirt is purely radiative.

The purple line, which is the wall temperature without film cooling, is looked at to determine the attachment point of the skirt. The purple line drops below the red line, or the failure temperature of the skirt [15], at an area ratio of 15, so the skirt can safely be attached at an area ratio of 20. Since the ISP penalty for a lower O/F ratio is very slight, an O/F ratio of 0.50 is chosen for film cooling to minimize wall temperature.

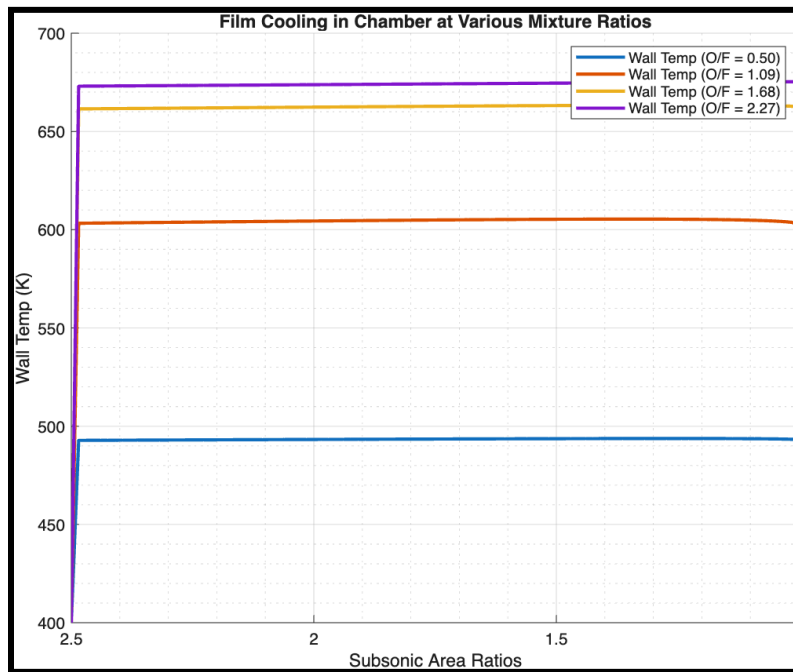


Figure 13. Film Cooling in Chamber at Various Mixture Ratios

The chamber wall temperatures are much less crucial, as there is no viscous heating (velocity is approximately zero in the chamber). An O/F ratio of 0.50 is still the best choice after investigating Figure 13.

## VIII. CAD Model

### **Nozzle + Chamber and Skirt:**

The nozzle, chamber, and skirt coordinates were imported as .txt files to SolidWorks, where the data was used to create a curve, defining the geometry to be revolved into the 3D model.

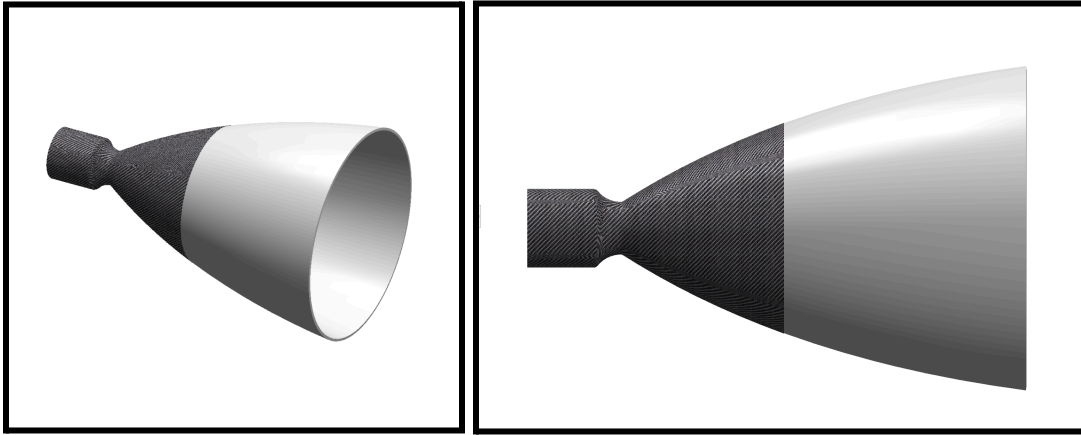


Figure 14. Views of the Nozzle + Chamber and Skirt

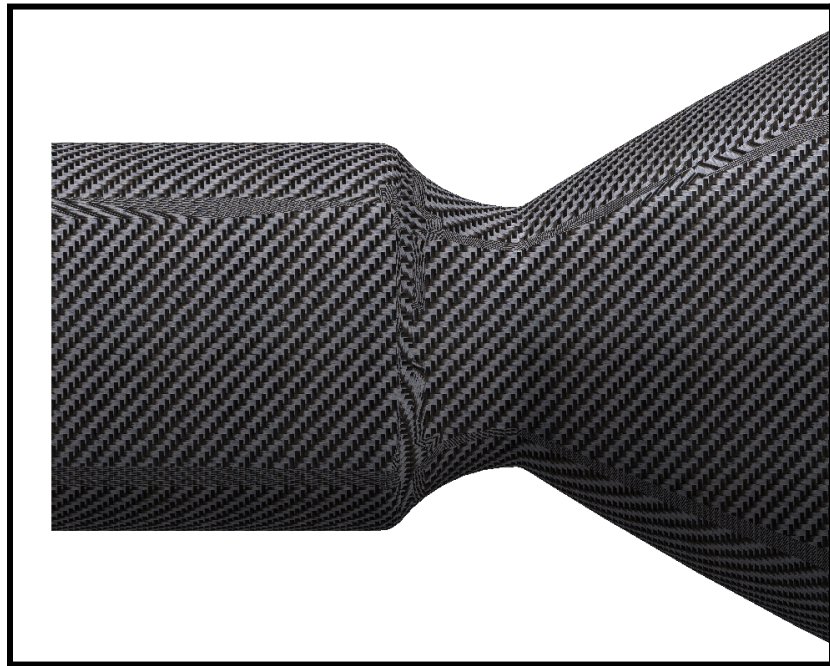


Figure 15. Close-up of Carbon Fiber

**Injector:**

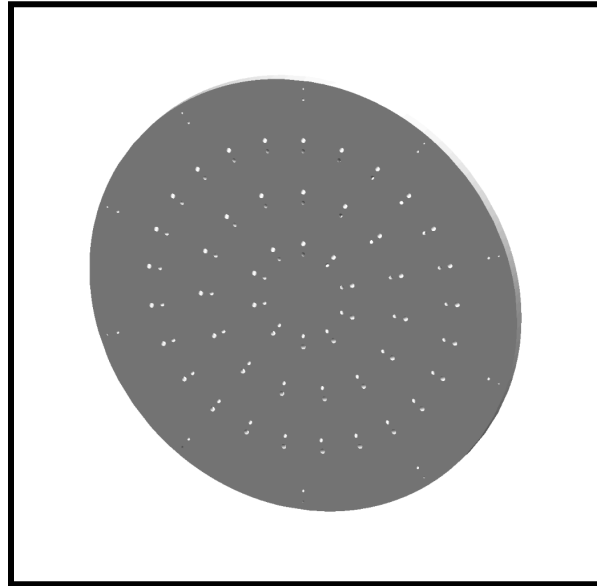


Figure 16. Injector Overview

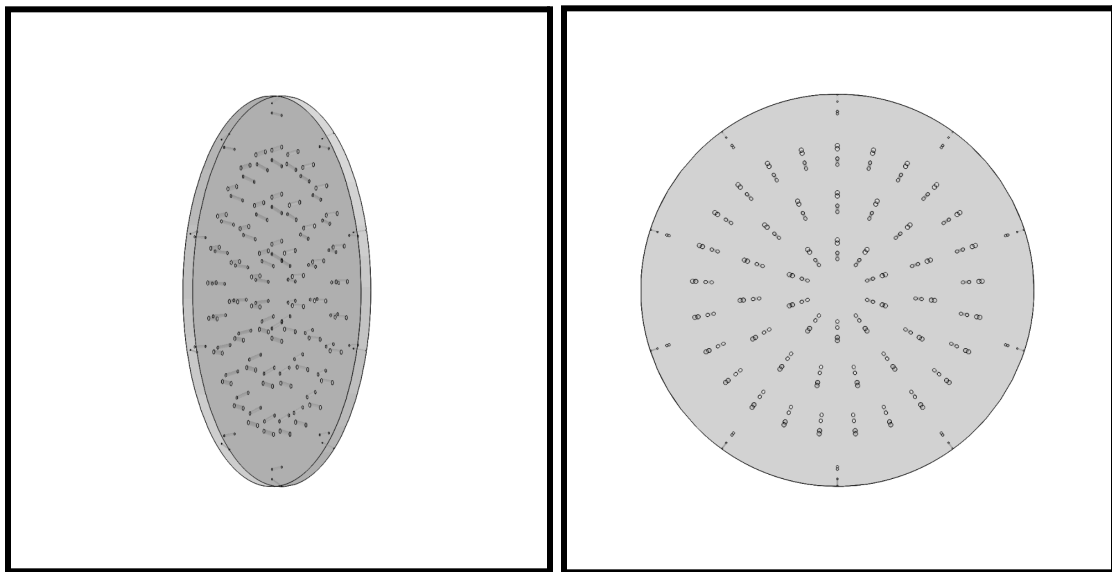


Figure 17. Transparent Views

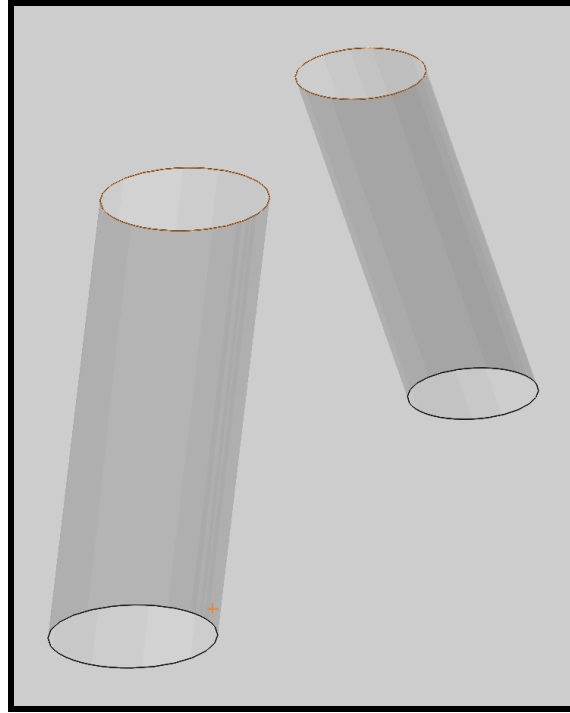


Figure 18. Single Doublet

## IX. Bench Marking

### Reference Engines: Space Shuttle OMS Engine (AJ10-190) and Apollo LMDE

Two heritage engines are used as benchmarks for this design. The primary benchmark is the Orbital Maneuvering System (OMS) Engine [16], developed by Aerojet, as it shares the identical propellant combination of MMH and NTO. The secondary benchmark is the Lunar Module Descent Engine (LMDE) [17], developed by TRW, as it performed the identical mission of lunar descent and ascent. These two engines frame this design from both a propellant standpoint and a mission application standpoint.

|                            | <b>OMS Engine</b> | <b>LMDE</b>             | <b>This Design</b>      |
|----------------------------|-------------------|-------------------------|-------------------------|
| <b>Propellant</b>          | MMH/NTO           | Aerozine-50/NTO         | MMH/NTO                 |
| <b>Feed System</b>         | Pressure-fed, He  | Pressure-fed, He        | Pressure-fed, He        |
| <b>Thrust (per engine)</b> | 26.7 kN           | 46.7 kN                 | 43 kN                   |
| <b>Vacuum ISP</b>          | 316 s             | 305 s                   | 339 s                   |
| <b>O/F Ratio</b>           | 1.65              | 1.6                     | 2.27                    |
| <b>Chamber Pressure</b>    | 125 psia.         | 103.4 psia              | 100 psia                |
| <b>Expansion Ratio</b>     | 55:1              | 47.5:1 (early design)   | 50:1                    |
| <b>Chamber Cooling</b>     | Regenerative      | Ablative + film cooling | Ablative + film cooling |
| <b>Nozzle Skirt</b>        | Columbium alloy   | C-103 columbium         | C-103 columbium         |
| <b>Throttling</b>          | Fixed             | 10%-100%                | 43%-100%                |

Table 6. Engine comparisons

### Discussion:

The most important benchmark result is the ISP comparison. The OMS engine achieves 316 s with MMH/NTO at O/F = 1.65, while this design achieves 339 s at O/F = 2.27. This can be explained due to higher ISP penalties because of the regenerative cooling system utilized by the OMS engine. Also, the ISP calculated for this design is a very rough, preliminary estimate. More

heat transfer analysis can be done in future design iterations, with possible higher percentages of mass flow being diverted to film cooling, thus bringing down the ISP. Also, actual performance will never reach the theoretical maximum performance simulated via computer software.

The chamber pressure comparison is another important benchmark. The normal chamber pressure during an OMS burn is approximately 125 psia. This design operates at 100 psia, which is 20% lower. This was a deliberate choice to minimize propellant tank feed pressure and wall thickness, justified by the CEA sweep in section two, which showed ISP is nearly insensitive to chamber pressure over the 100–200 psia range. The OMS engine operates at slightly higher  $P_c$  for a different reason — regenerative cooling requires higher pressure drop across the cooling channels, driving the feed pressure and therefore  $P_c$  upward. Since this design does not use regenerative cooling, the lower 100 psia is both sufficient and optimal.

The nozzle architecture is essentially identical across both benchmarks and this design — all three use a columbium alloy radiation-cooled skirt for the nozzle extension. This similarity across three independently developed engines spanning 1969 to 1981 is strong evidence that columbium radiation cooling is the correct engineering choice for hypergolic engines in this thrust class operating in vacuum.

The O/F ratio difference between the OMS engine (1.65) and this design (2.27) is another source of discussion. The OMS engine was designed at  $O/F = 1.65$ , which is also the ratio used by the LMDE. This design's CEA optimization in section two identified 2.27 as the ISP-maximizing ratio for MMH/NTO at 100 psia. This difference may reflect that the heritage engines were designed with margin away from the optimal O/F to reduce combustion temperature and heat loads. Moving forward, the O/F difference is an important metric that future work on this design should investigate further.

Overall, the comparison between the engines validates this design. Chamber pressure, expansion ratio, nozzle material, feed architecture, and delivered ISP are all very similar across the board, displaying that the preliminary design parameters are physically reasonable and consistent with proven engineering practice.

## X. Appendix / Citations / Resources used

### Textbooks (General References):

S. D. Heister, W. E. Anderson, T. Pourpoint, and R. J. Sardeshmukh, *Rocket Propulsion*. Cambridge, UK: Cambridge University Press, 2019.

D. K. Huzel and D. H. Huang, *Modern Engineering for Design of Liquid-Propellant Rocket Engines*, rev. ed. Washington, DC: AIAA, 1992.

G. P. Sutton and O. Biblarz, *Rocket Propulsion Elements*, 8th ed. Hoboken, NJ: Wiley, 2010.

### Software:

NASA Glenn Research Center, "Chemical Equilibrium with Applications (CEA)," NASA, 2023. [Online]. Available: <https://cearun.grc.nasa.gov/>

The MathWorks Inc., "MATLAB R2025a," Natick, MA, USA: The MathWorks Inc., 2024. [Online]. Available: <https://www.mathworks.com>

Dassault Systèmes, SOLIDWORKS 2024, Waltham, MA, USA: Dassault Systèmes, 2024. [Online]. Available: <https://www.solidworks.com>

### Websites:

[1] M. Wade, "Astronautix Encyclopedia," Astronautix, 2019. [Online]. Available: <http://www.astronautix.com/>

[2] D. Arney et al., "Understanding the Lunar System Architecture Design Space," NASA NTRS, 2014. [Online]. Available: <https://ntrs.nasa.gov/api/citations/20140002734/downloads/20140002734.pdf>

[3] A. Hedayat et al., "Pressurization, Pneumatic, and Vent Subsystems of the X-34 Main Propulsion System," NASA NTRS, 1998. [Online]. Available: <https://ntrs.nasa.gov/api/citations/19980236000/downloads/19980236000.pdf>

[4] A. Helmenstine, "What Is the Most Ideal Gas?," ThoughtCo, Jan. 2019. [Online]. Available: <https://www.thoughtco.com/what-is-the-most-ideal-gas-607548>

- [5] NASA Science, "Weather on the Moon," NASA, 2023. [Online]. Available: <https://science.nasa.gov/moon/weather-on-the-moon/>
- [6] AEETHER, "The New Darling of the Future Automotive Industry: Inconel Alloy," *AEETHER*, 2024. [Online]. Available: <https://www.aeether.com/AEETHER/media/media-75/media.html>
- [7] AspireSpace, "Thrust Optimised Parabolic Nozzle," AspireSpace Technical Report, 2012. [Online]. Available: <http://www.aspirespace.org.uk/downloads/Thrust%20optimised%20parabolic%20nozzle.pdf>
- [8] Carbon Fiber Gear, "How Strong Is Carbon Fiber?," Carbon Fiber Gear, 2023. [Online]. Available: <https://carbonfiberglass.com/blogs/carbonfiber/how-strong-is-carbon-fiber>
- [9] Amardeep Steel, "Titanium Alloy Ti-6Al-4V Properties and Applications," Amardeep Steel Blog, 2022. [Online]. Available: <https://www.amardeepsteel.com/blog/titanium-alloys-6-4.html>
- [10] CBMM Niobium, "Thermomechanical Processing and Properties of Niobium Alloys," CBMM Technical Report, 2020. [Online]. Available: [https://niobium.tech/-/media/niobiumtech/attachments-biblioteca-tecnica/nt\\_thermomechanical-processing-and-properties-of-niobium-alloys.pdf](https://niobium.tech/-/media/niobiumtech/attachments-biblioteca-tecnica/nt_thermomechanical-processing-and-properties-of-niobium-alloys.pdf)
- [11] DragonPlate, "Carbon Fiber Thermal Conductivity and the Coefficient of Thermal Expansion," DragonPlate, 2023. [Online]. Available: <https://dragonplate.com/carbon-fiber-thermal-conductivity-and-the-coefficient-of-thermal-expansion>
- [12] S. Kurna et al., "Thermal Conductivity and Density of Phenolic Resin Composites," *PMC — National Library of Medicine*, 2022. [Online]. Available: <https://pmc.ncbi.nlm.nih.gov/articles/PMC9404460/>
- [13] Azista USA, "Phenolic Prepregs — Material Properties," Azista USA Composites, 2023. [Online]. Available: <https://www.azistausa.com/composites/prepregs/phenolic-prepregs>
- [14] O. Haidn et al., "Experimental Investigation of Film Cooling with Tangential Slot Injection in a LOX/CH<sub>4</sub> Subscale Rocket Combustion Chamber," *Acta Astronautica*, 2012. [Online]. Available: <https://www.researchgate.net/profile/Oskar-Haidn-3/publication/225004887>

[15] Y. Zhang et al., "Thermal Barrier Coatings for High Temperature Applications," Surface and Coatings Technology, vol. 479, 2024. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0257897224001403>

[16] M. Wade, "OMS Engine (AJ10-190)," Astronautix, 2019. [Online]. Available: <http://www.astronautix.com/o/ome.html>

[17] R. O. House, "Lunar Module Descent Engine — TRW Heritage," Engineering History, 2010. [Online]. Available: <https://enginehistory.org/Rockets/RPE09.42/RPE09.42.shtml>